PG – 261

IV Semester M.Sc. Degree Examination, June/July 2014
(RNS) (2012-13 and Onwards)
MATHEMATICS
M – 403 (C) : Theory of Numbers

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer any five questions.*
ii) All questions carry equal marks.

1. a) Establish a relation connecting the Euler's function and the Möbius function.

   b) Derive a product formula for the Euler's function.

   c) Prove that $\varphi(n) > n/6$ for all n with atmost 8 distinct prime factors. **(6+6+4)**

2. a) Prove that the set of all arithmetical functions f with $f(1) \neq 0$ forms an abelian group under Dirichlet multiplication.

   b) Prove that the Möbius function $\mu(n)$ is the sum of the primitive n^{th} roots of unity. **(10+6)**

3. a) If f and g are multiplicative, then show that their Dirichlet multiplication $f * g$ is also multiplicative.

   b) Prove that a multiplicative function f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n), n \geq 1$.

   c) Prove that $\sum_{d/n} d\,\mu(d) = \pi_{p/n}(1-p)$, p – prime. **(5+6+5)**

4. a) State and prove Wilson's theorem. Is the converse of Wilson's theorem true ? Justify.

   b) Find all positive integers n for which $(n-1)! + 1$ is a power of n.

   c) State Chinese remainder theorem. Show that the set of lattice points in the plane visible from the origin contains arbitrarily large square gaps. **(7+3+6)**

P.T.O.

5\. a) Define Legendre's symbol (n/p). Let p be an odd prime. Prove that $(n/p) \equiv n^{(p-1)/2} \pmod{p}$.

b) For every odd prime p, show that $(2/p) = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod{8} \\ -1 & \text{if } p \equiv \pm 3 \pmod{8} \end{cases}$

c) State quadratic reciprocity law. Determine whether 888 is a quadratic residue or non-residue of the prime 1999. (6+5+5)

6\. a) Given $m \geq 1$, where m is not of the form $m = 1, 2, 4, p^{\alpha}$ or $2p^{\alpha}$ where p is an odd prime and $\alpha \geq 1$, prove that there are no primitive roots mod m.

b) Prove that the sum of the primitive roots mod p is congruent to $\mu(p-1)$ mod p. (12+4)

7\. a) Define the partition function p (n). Prove that $\sum_{n=0}^{\infty} p(n) q^n = \frac{1}{(q;q)_{\infty}}, |q| < 1$

b) Prove that $p(7m+5) \equiv 0 \pmod{7}$. (8+8)

8\. State and prove Rogers-Ramanujan identities. 16
